Normal Distributions

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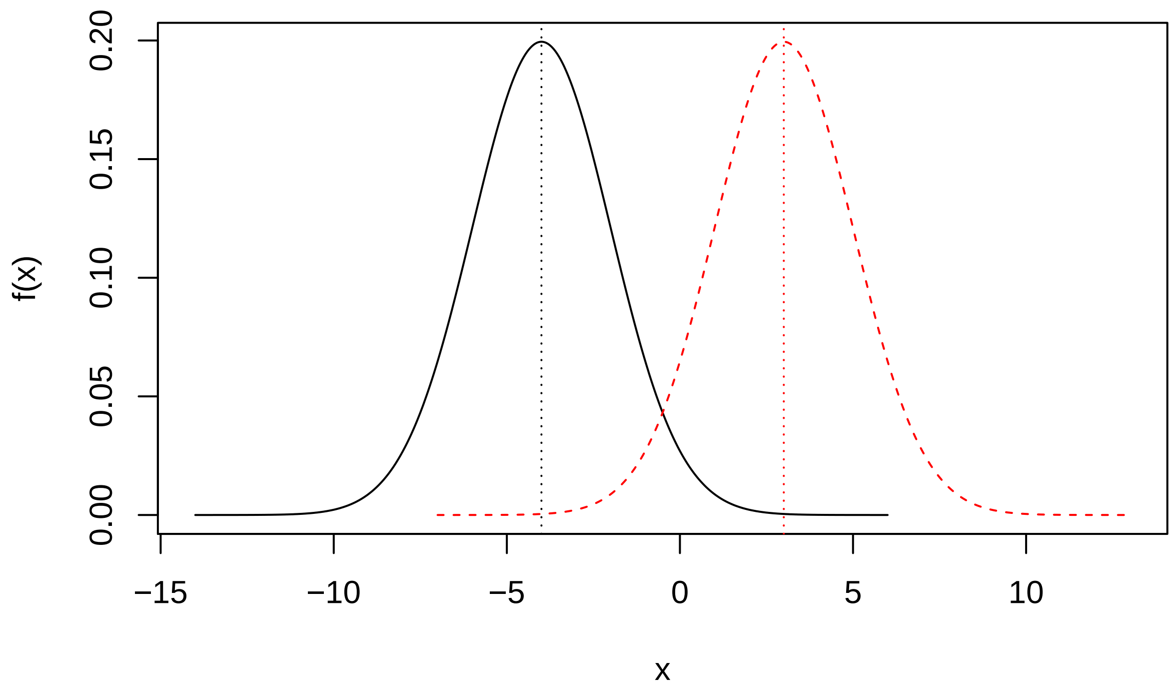
Normal

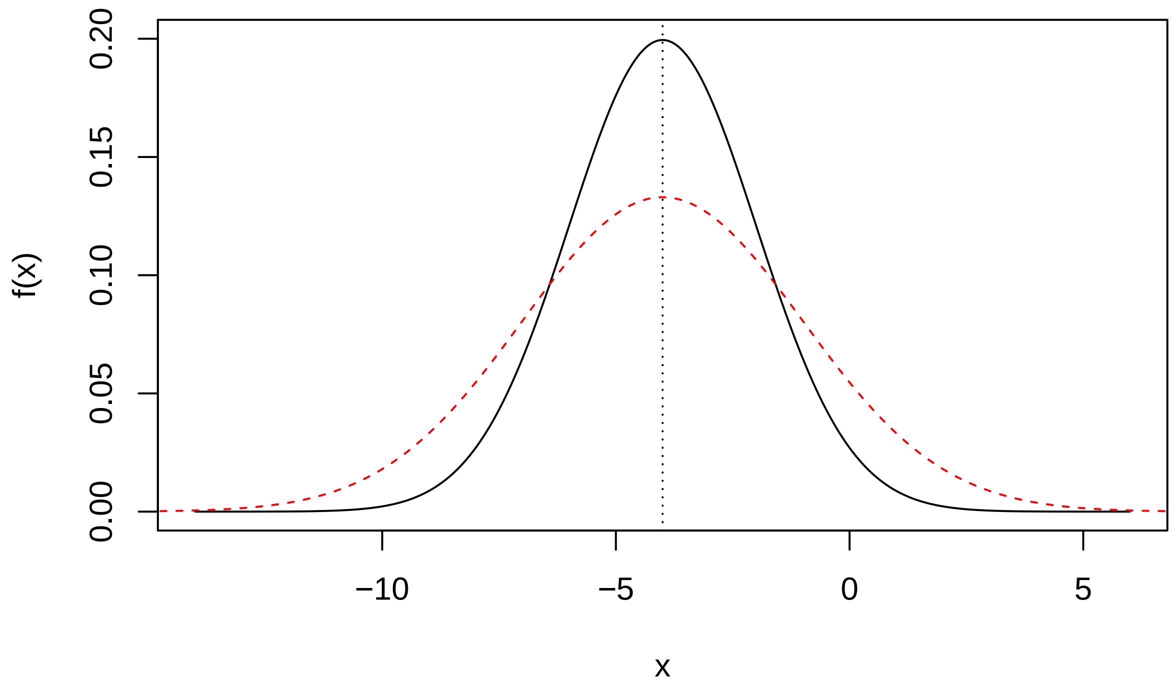
We can use the base plot functions in R to create a plot of the pdf for a normal random variable, X, with mean, μ, and variance, σ^2 — that is, $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$.

```r
z <- seq(-5, 5, by=0.01)
mu1 <- -4
mu2 <- 3
sigma1 <- 2
sigma2 <- 3
x1 <- mu1 + z*sigma1
x2 <- mu2 + z*sigma1
plot(x1, dnorm(x1, mu1, sigma1), lty=1, col=1, type="l",
     xlab="x", ylab="f(x)", xlim=range(c(x1, x2)))
lines(x2, dnorm(x2, mu2, sigma1), lty=2, col=2)
abline(v=mu1, col=1, lty=3)
abline(v=mu2, col=2, lty=3)
```



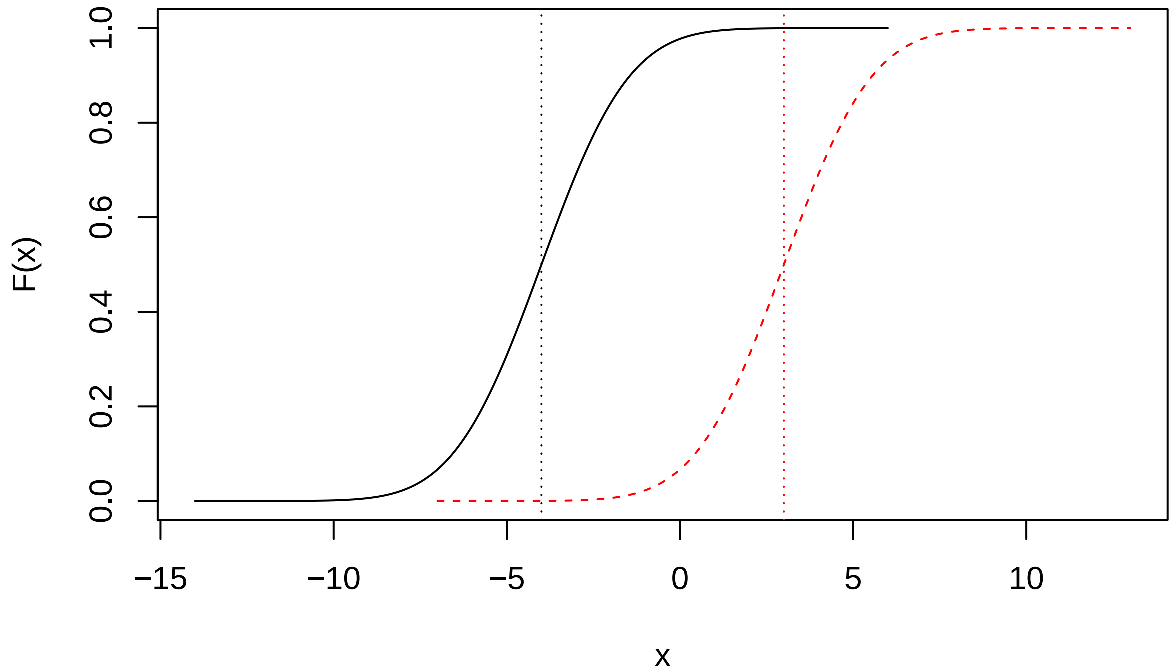

```r
x1 <- mu1 + z*sigma1
x2 <- mu1 + z*sigma2
plot(x1, dnorm(x1, mu1, sigma1), lty=1, col=1, type="l",
     xlab="x", ylab="f(x)", ylim=c(0, 0.2))
```

```r
lines(x2, dnorm(x2, mu1, sigma2), lty=2, col=2)
abline(v=mu1, col=1, lty=3)
```

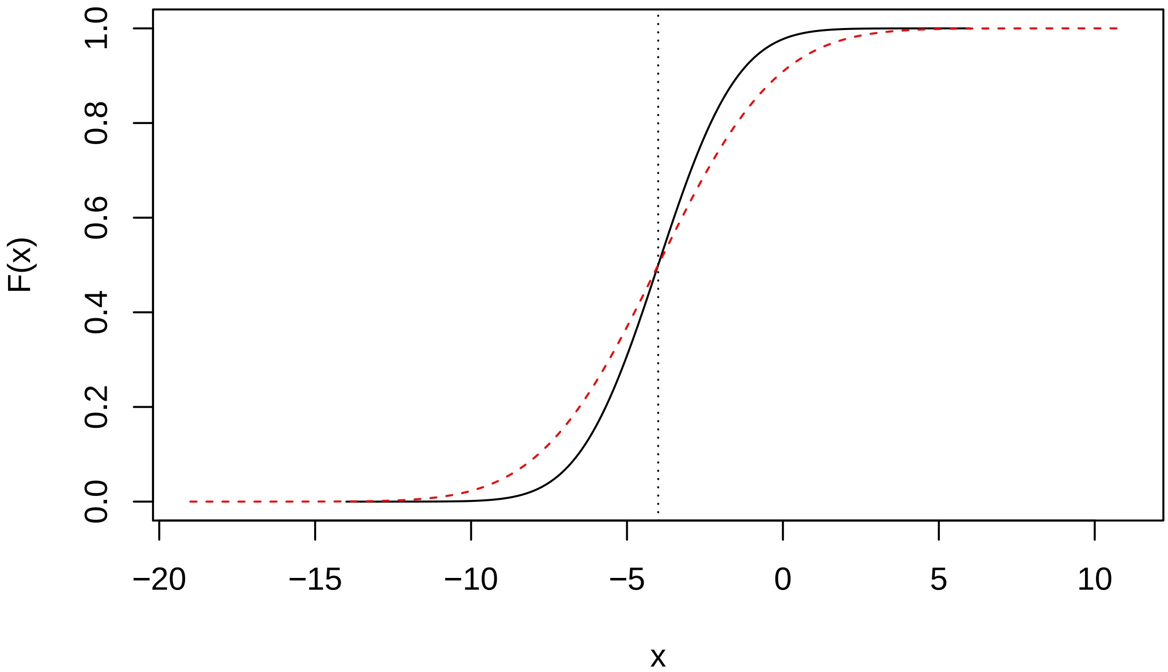


The CDF may be plotted analogously.

```r
z <- seq(-5, 5, by=0.01)
mu1 <- -4
mu2 <- 3
sigma1 <- 2
sigma2 <- 3
x1 <- mu1 + z*sigma1
x2 <- mu2 + z*sigma1
plot(x1, pnorm(x1, mu1, sigma1), lty=1, col=1, type="l",
     xlab="x", ylab="F(x)", xlim=range(c(x1, x2)))
lines(x2, pnorm(x2, mu2, sigma1), lty=2, col=2)
abline(v=mu1, col=1, lty=3)
abline(v=mu2, col=2, lty=3)
```



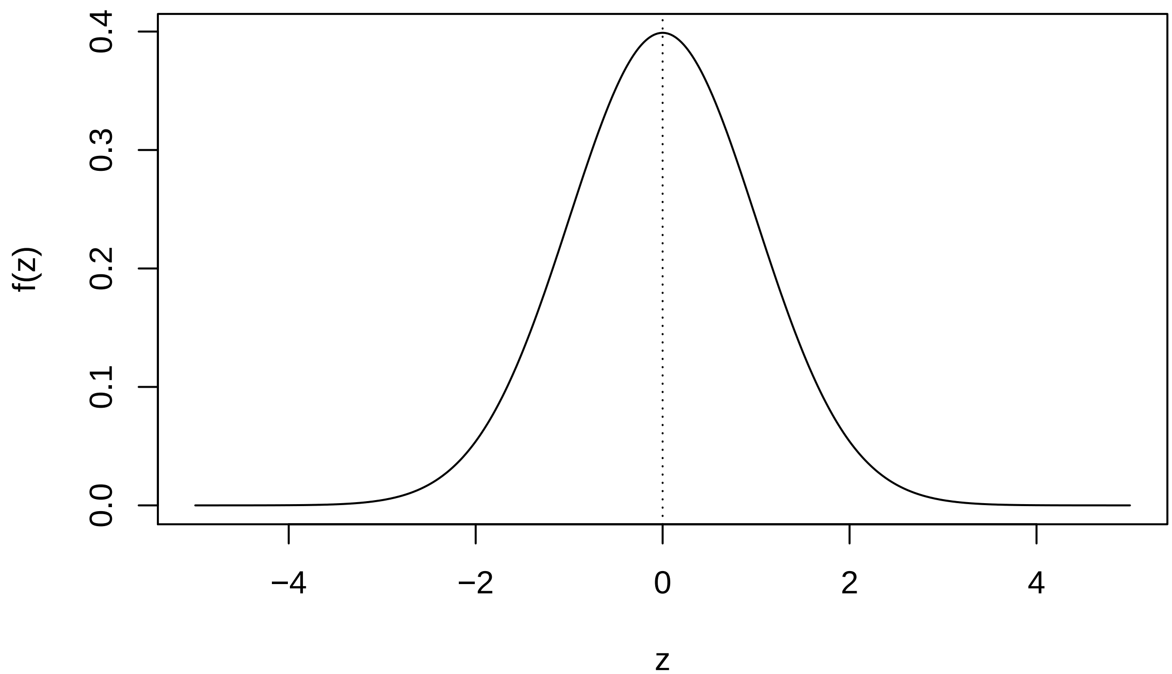

```r
x1 <- mu1 + z*sigma1
x2 <- mu1 + z*sigma2
plot(x1, pnorm(x1, mu1, sigma1), lty=1, col=1, type="l",
     xlab="x", ylab="F(x)", xlim=range(c(x1, x2)))
lines(x2, pnorm(x2, mu1, sigma2), lty=2, col=2)
abline(v=mu1, col=1, lty=3)
```



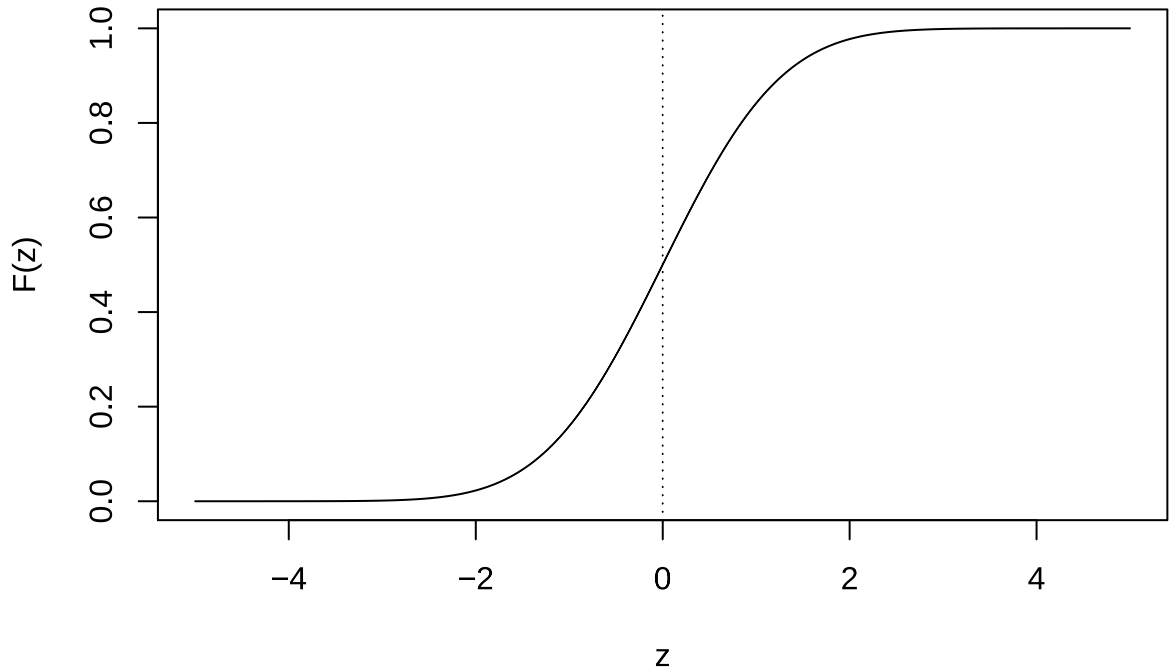
Standard Normal

The special case of the normal is actually a $Z \sim \mathrm{N}(0, 1)$.

```
plot(z, dnorm(z), type="l", xlab="z", ylab="f(z)")
abline(v=0, lty=3)
```




```r
plot(z, pnorm(z), type="l", xlab="z", ylab="F(z)")
abline(v=0, lty=3)
```



Since all normals can be transformed to the standard normal, we need just a single table. Software works in the same way — by transformation to and from the standard normal. We look at some values and their probabilities.

```
z <- c((-3):3)
rbind(z,pnorm(z))
```

```
##          [,1]        [,2]       [,3] [,4]      [,5]      [,6]      [,7]
## z -3.000000000 -2.00000000 -1.0000000  0.0 1.0000000 2.0000000 3.0000000
##    0.001349898  0.02275013  0.1586553  0.5 0.8413447 0.9772499 0.9986501
```

```
x <- mu1 + sigma1*z
rbind(x,pnorm(x, mu1, sigma1))
```

```
##           [,1]        [,2]       [,3] [,4]       [,5]      [,6]      [,7]
## x -10.000000000 -8.00000000 -6.0000000 -4.0 -2.0000000 0.0000000 2.0000000
##     0.001349898  0.02275013  0.1586553  0.5  0.8413447 0.9772499 0.9986501
```

```
pnorm(x, mu1, sigma1) %*% c(0,-1,0,0,0,1,0)
```

```
##          [,1]
## [1,] 0.9544997
```

```
q <- c(0.005, 0.025, 0.05, 0.95, 0.975, 0.995)
rbind(q, qnorm(q))
```

```
##       [,1]      [,2]      [,3]     [,4]     [,5]     [,6]
## q  0.005000  0.025000  0.050000 0.950000 0.975000 0.995000
##   -2.575829 -1.959964 -1.644854 1.644854 1.959964 2.575829
```

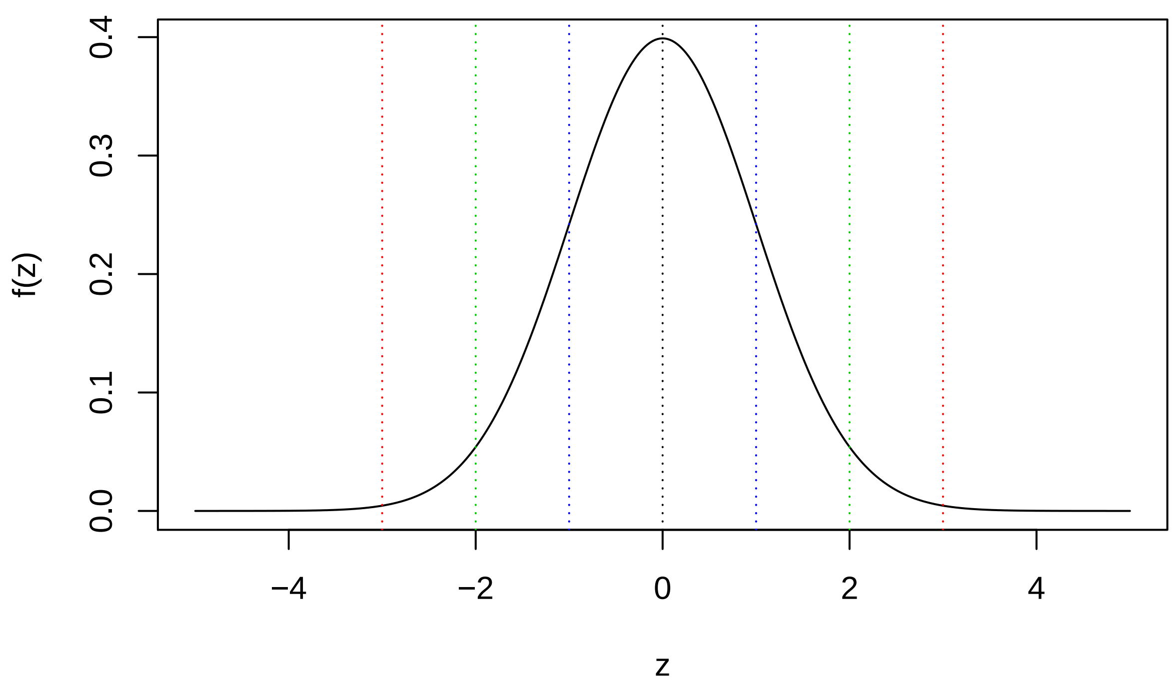
```
rbind(q, qnorm(q, mu1, sigma1))
```

```
##         [,1]      [,2]      [,3]       [,4]        [,5]     [,6]
## q  0.005000  0.025000  0.050000  0.9500000  0.97500000 0.995000
##   -9.151659 -7.919928 -7.289707 -0.7102927 -0.08007203 1.151659
```

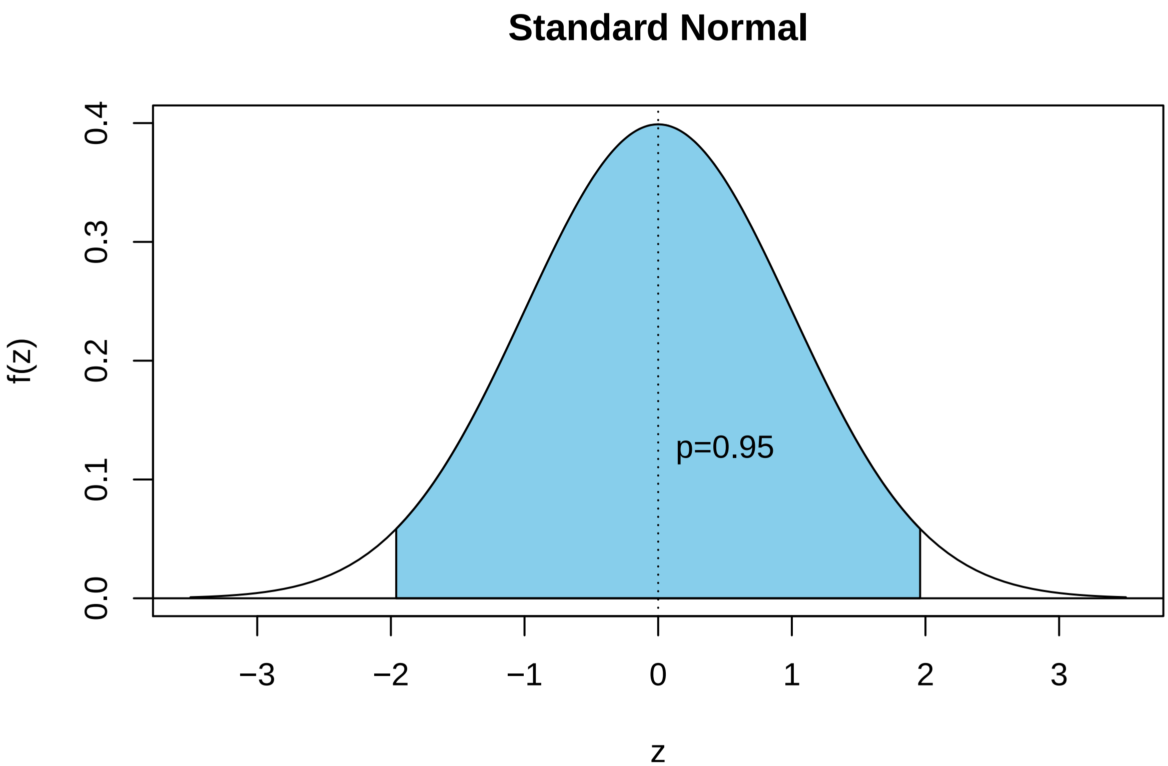
Empirical Rule

The empirical rule gives approximate probabilities for a few "interesting" points. Consider $\mu \pm k\sigma$ for $k = 1, 2, 3$. For normal data we get:

```
z <- seq(-5, 5, by=0.01)
plot(z, dnorm(z), type="l", xlab="z", ylab="f(z)")
abline(v=c(0,-3,-2,-1,1,2,3), lty=3, col=c(1,2:4,4:2))
```




```
cord.x <- c(-1.96,seq(-1.96,1.96,0.01),1.96)
cord.y <- c(0,dnorm(seq(-1.96,1.96,0.01)),0)
curve(dnorm(x,0,1),xlim=c(-3.5,3.5),
      main='Standard Normal', ylab="f(z)", xlab="z")
polygon(cord.x,cord.y,col='skyblue')
abline(v=0, lty=3)
abline(h=0, lty=1)
text(0.5,0.125,"p=0.95")
```



Standard Normal
f(z)
0.0
0.1
0.2
0.3
0.4
p=0.95
−3
−2
−1
0
1
2
3
z